

NAG C Library Function Document

nag_zherfs (f07mvc)

1 Purpose

nag_zherfs (f07mvc) returns error bounds for the solution of a complex Hermitian indefinite system of linear equations with multiple right-hand sides, $AX = B$. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```
void nag_zherfs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer nrhs,
  const Complex a[], Integer pda, const Complex af[], Integer pdaf,
  const Integer ipiv[], const Complex b[], Integer pdb, Complex x[],
  Integer pdx, double ferr[], double berr[], NagError *fail)
```

3 Description

nag_zherfs (f07mvc) returns the backward errors and estimated bounds on the forward errors for the solution of a complex Hermitian indefinite system of linear equations with multiple right-hand sides $AX = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of nag_zherfs (f07mvc) in terms of a single right-hand side b and solution x .

Given a computed solution x , the function computes the *component-wise backward error* β . This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b \\ |\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the f07 Chapter Introduction.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType

Input

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

- 2: **uplo** – Nag_UploType *Input*
On entry: indicates whether the upper or lower triangular part of A is stored and how A has been factorized, as follows:
 if **uplo** = **Nag_Upper**, then the upper triangular part of A is stored and A is factorized as $PUDU^H P^T$, where U is upper triangular;
 if **uplo** = **Nag_Lower**, then the lower triangular part of A is stored and A is factorized as $PLDL^H P^T$, where L is lower triangular.
Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.
- 3: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.
- 4: **nrhs** – Integer *Input*
On entry: r , the number of right-hand sides.
Constraint: **nrhs** ≥ 0 .
- 5: **a**[dim] – const Complex *Input*
Note: the dimension, dim , of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.
 If **order** = **Nag_ColMajor**, the (i, j) th element of the matrix A is stored in **a**[($j - 1$) $\times$ **pda** + $i - 1$] and
 if **order** = **Nag_RowMajor**, the (i, j) th element of the matrix A is stored in **a**[($i - 1$) $\times$ **pda** + $j - 1$].
On entry: the n by n original Hermitian matrix A as supplied to nag_zhetrf (f07mrc).
- 6: **pda** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **a**.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.
- 7: **af**[dim] – const Complex *Input*
Note: the dimension, dim , of the array **af** must be at least $\max(1, \mathbf{pdaf} \times \mathbf{n})$.
On entry: details of the factorization of A , as returned by nag_zhetrf (f07mrc).
- 8: **pdaf** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **af**.
Constraint: **pdaf** $\geq \max(1, \mathbf{n})$.
- 9: **ipiv**[dim] – const Integer *Input*
Note: the dimension, dim , of the array **ipiv** must be at least $\max(1, \mathbf{n})$.
On entry: details of the interchanges and the block structure of D , as returned by nag_zhetrf (f07mrc).
- 10: **b**[dim] – const Complex *Input*
Note: the dimension, dim , of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.
 If **order** = **Nag_ColMajor**, the (i, j) th element of the matrix B is stored in **b**[($j - 1$) $\times$ **pdb** + $i - 1$] and
 if **order** = **Nag_RowMajor**, the (i, j) th element of the matrix B is stored in **b**[($i - 1$) $\times$ **pdb** + $j - 1$].
On entry: the n by r right-hand side matrix B .

- 11: **pdb** – Integer *Input*
On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.
Constraints:
 if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, n)$;
 if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, nrhs)$.
- 12: **x**[*dim*] – Complex *Input/Output*
Note: the dimension, *dim*, of the array **x** must be at least $\max(1, \mathbf{pdx} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdx} \times n)$ when **order** = **Nag_RowMajor**.
 If **order** = **Nag_ColMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*j* – 1) $\times$ **pdx** + *i* – 1] and if **order** = **Nag_RowMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*i* – 1) $\times$ **pdx** + *j* – 1].
On entry: the *n* by *r* solution matrix *X*, as returned by nag_zhetrs (f07msc).
On exit: the improved solution matrix *X*.
- 13: **pdx** – Integer *Input*
On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **x**.
Constraints:
 if **order** = **Nag_ColMajor**, **pdx** $\geq \max(1, n)$;
 if **order** = **Nag_RowMajor**, **pdx** $\geq \max(1, nrhs)$.
- 14: **ferr**[*dim*] – double *Output*
Note: the dimension, *dim*, of the array **ferr** must be at least $\max(1, nrhs)$.
On exit: **ferr**[*j* – 1] contains an estimated error bound for the *j*th solution vector, that is, the *j*th column of *X*, for *j* = 1, 2, ..., *r*.
- 15: **berr**[*dim*] – double *Output*
Note: the dimension, *dim*, of the array **berr** must be at least $\max(1, nrhs)$.
On exit: **berr**[*j* – 1] contains the component-wise backward error bound β for the *j*th solution vector, that is, the *j*th column of *X*, for *j* = 1, 2, ..., *r*.
- 16: **fail** – NagError * *Output*
 The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = *<value>*.

Constraint: **n** ≥ 0 .

On entry, **nrhs** = *<value>*.

Constraint: **nrhs** ≥ 0 .

On entry, **pda** = *<value>*.

Constraint: **pda** > 0 .

On entry, **pda** = *<value>*.

Constraint: **pda** > 0 .

On entry, **pdb** = *<value>*.

Constraint: **pdb** > 0 .

On entry, **pdx** = $\langle value \rangle$.
 Constraint: **pdx** > 0.

NE_INT_2

On entry, **pda** = $\langle value \rangle$, **n** = $\langle value \rangle$.
 Constraint: **pda** $\geq$ max(1, **n**).

On entry, **pda** = $\langle value \rangle$, **n** = $\langle value \rangle$.
 Constraint: **pda** $\geq$ max(1, **n**).

On entry, **pdb** = $\langle value \rangle$, **n** = $\langle value \rangle$.
 Constraint: **pdb** $\geq$ max(1, **n**).

On entry, **pdb** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.
 Constraint: **pdb** $\geq$ max(1, **nrhs**).

On entry, **pdx** = $\langle value \rangle$, **n** = $\langle value \rangle$.
 Constraint: **pdx** $\geq$ max(1, **n**).

On entry, **pdx** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.
 Constraint: **pdx** $\geq$ max(1, **nrhs**).

NE_SINGULAR

The block diagonal matrix D is exactly singular.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ real floating-point operations. Each step of iterative refinement involves an additional $24n^2$ real operations. At most 5 steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real operations.

The real analogue of this function is nag_dsyrfs (f07mhc).

9 Example

To solve the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 7.79 & + & 5.48i & -35.39 & + & 18.01i \\ -0.77 & - & 16.05i & 4.23 & - & 70.02i \\ -9.58 & + & 3.88i & -24.79 & - & 8.40i \\ 2.98 & - & 10.18i & 28.68 & - & 39.89i \end{pmatrix}.$$

Here A is Hermitian indefinite and must first be factorized by nag_zhetrf (f07mrc).

9.1 Program Text

```
/* nag_zherfs (f07mvc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, n, nrhs, pda, pdaf, pdb, pdx;
    Integer ferr_len, berr_len;
    Integer exit_status=0;
    Nag_UploType uplo_enum;
    NagError fail;
    Nag_OrderType order;

    /* Arrays */
    Integer *ipiv=0;
    char uplo[2];
    Complex *a=0, *af=0, *b=0, *x=0;
    double *berr=0, *ferr=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define AF(I,J) af[(J-1)*pdaf + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
#define X(I,J) x[(J-1)*pdx + I - 1]
    order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define AF(I,J) af[(I-1)*pdaf + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
#define X(I,J) x[(I-1)*pdx + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);
    Vprintf("f07mvc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n] ");
    Vscanf("%ld%ld%*[^\\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
    pda = n;
    pdaf = n;

```

```

    pdb = n;
    pdx = n;
#else
    pda = n;
    pdaf = n;
    pdb = nrhs;
    pdx = nrhs;
#endif

    ferr_len = nrhs;
    berr_len = nrhs;

/* Allocate memory */
if ( !(ipiv = NAG_ALLOC(n, Integer)) ||
    !(a = NAG_ALLOC(n * n, Complex)) ||
    !(af = NAG_ALLOC(n * n, Complex)) ||
    !(b = NAG_ALLOC(n * nrhs, Complex)) ||
    !(x = NAG_ALLOC(n * nrhs, Complex)) ||
    !(berr = NAG_ALLOC(berr_len, double)) ||
    !(ferr = NAG_ALLOC(ferr_len, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file, and copy A to AF and B to X */

Vscanf(" ' %ls '%*[\n] ", uplo);
if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
else
{
    Vprintf("Unrecognised character for Nag_UploType type\n");
    exit_status = -1;
    goto END;
}
if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
    }
    Vscanf("%*[\n] ");
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
    }
    Vscanf("%*[\n] ");
}
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
}
Vscanf("%*[\n] ");
/* Copy A to AF and B to X */
if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
        {
            AF(i,j).re = A(i,j).re;

```

```

        AF(i,j).im = A(i,j).im;
    }
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
        {
            AF(i,j).re = A(i,j).re;
            AF(i,j).im = A(i,j).im;
        }
    }
}
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
    {
        X(i,j).re = B(i,j).re;
        X(i,j).im = B(i,j).im;
    }
}
/* Factorize A in the array AF */
f07mrc(order, uplo_enum, n, af, pdaf, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07mrc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Compute solution in the array X */
f07msc(order, uplo_enum, n, nrhs, af, pdaf, ipiv, x, pdx,
        &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07msc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Improve solution, and compute backward errors and */
/* estimated bounds on the forward errors */
f07mvc(order, uplo_enum, n, nrhs, a, pda, af, pdaf, ipiv,
        b, pdb, x, pdx, ferr, berr, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07mvc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print solution */
x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, x, pdx,
        Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels,
        0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from x04dbc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
Vprintf("\nBackward errors (machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", berr[j-1], j%4 == 0 ? "\n": " ");
Vprintf("\nEstimated forward error bounds "
        "(machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", ferr[j-1], j%4 == 0 ? "\n": " ");
Vprintf("\n");
END:
if (ipiv) NAG_FREE(ipiv);
if (a) NAG_FREE(a);

```

```

    if (af) NAG_FREE(af);
    if (b) NAG_FREE(b);
    if (x) NAG_FREE(x);
    if (berr) NAG_FREE(berr);
    if (ferr) NAG_FREE(ferr);
    return exit_status;
}

```

9.2 Program Data

f07mvc Example Program Data

```

    4 2                                     :Values of N and NRHS
    'L'                                   :Value of UPLO
    (-1.36, 0.00)
    ( 1.58,-0.90) (-8.87, 0.00)
    ( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
    ( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A
    ( 7.79,  5.48) (-35.39, 18.01)
    (-0.77,-16.05) (  4.23,-70.02)
    (-9.58,  3.88) (-24.79, -8.40)
    ( 2.98,-10.18) ( 28.68,-39.89)           :End of matrix B

```

9.3 Program Results

f07mvc Example Program Results

```

Solution(s)
           1                2
1  ( 1.0000,-1.0000)  ( 3.0000,-4.0000)
2  (-1.0000, 2.0000) (-1.0000, 5.0000)
3  ( 3.0000,-2.0000) ( 7.0000,-2.0000)
4  ( 2.0000, 1.0000) (-8.0000, 6.0000)

Backward errors (machine-dependent)
    9.0e-17    5.8e-17
Estimated forward error bounds (machine-dependent)
    2.6e-15    3.0e-15

```
